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Report Title

Final Report: Topology and Foundations of Quantum Algorithms

ABSTRACT

This project developed new results in the topology and foundations of quantum algorithms. It made major developments in the program for topological quantum field computation, including the design of substantially simplified lattice models with topological phases, and new insights into the universality classes of such models. Lattice gas simulations of various classical and quantum systems were investigated. In addition, several new algorithms for structured search, concept learning, and image processing were discovered.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

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Graduate Students

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1. Foreword

Over the past decade or so, it has become clear that serious scientific and engineering obstacles will have to be overcome to build a scalable quantum computer. This will require substantial commitment—of personnel and funding. Just as it was in the initial development of electronic computers, the justification for such a large scale effort must be applications. In the current context this means additional problems, beyond factoring, which can be solved more efficiently with a quantum computer.

The exponential size of Hilbert space, interference, and entanglement are all properties of quantum systems which are known to be useful for various quantum information processing tasks. The goal of our project *Topology and Foundations of Quantum Algorithms* (DAAD19-01-1-0520) was to deepen this understanding to the point where we can develop new quantum algorithms, especially in the context of topological quantum field computation. It was a relatively large theoretical project supporting four principal investigators: Michael Freedman (Microsoft), David Meyer (UCSD), Nolan Wallach (UCSD) and Zhenghan Wang (Indiana) for three years, no-cost-extended to four. That the project succeeded is demonstrated by our development of several new quantum algorithms (described briefly in §3 and in detail in the 19 publications listed in §4), by the formation of Microsoft Station Q at the UCSB California NanoSystems Institute, and by having spun off three related projects: In a project funded by the Division of Electrical, Communications & Cyber Systems within the NSF's Engineering Directorate, Meyer has been collaborating with Markus Hunziker and Mitch Rothstein (Georgia) on quantum learning algorithms. Also DARPA's Defense Sciences Office funded a seedling program in which Meyer collaborated with a group at HP labs to investigate further properties of quantum games. And one of Meyer's graduate students, Yi-Kai Liu, is supported by an ARO Quantum Computing Graduate Fellowship; he will graduate this year, and has accepted a postdoctoral position at Caltech.

The results of this project have been presented in talks in mathematics, physics, chemistry and computer science departments at universities and government labs in the U. S., Argentina, Canada, France, Hong Kong, the Netherlands, Singapore and Switzerland, and at conferences in the U. S., Canada, China, Italy, Scotland and Singapore, and have been reported in the popular science press. In addition, Freedman and Wang helped organize the AIM Workshop *Topological Phases in Condensed Matter Physics* (September 2003) and Meyer helped organize an ARO workshop on *New Directions in Quantum Algorithms* (December 2004)

2. Problem statement

The underlying theme of our project is that further progress toward quantum computation must take advantage of what physics provides. The challenge is to understand more precisely the quantum mechanical resources available for computation—to determine when and how quantum phenomena yield advantages over classical computational models. We investigated these questions from three perspectives: First, since quantum systems have the potential to process information more efficiently than classical systems exactly to the extent that they cannot be efficiently simulated classically, we have considered simulation of physical systems, both classical and quantum, concentrating on limitations imposed by geometry (*e.g.*, lattice structure) and entanglement. Second, in the context of algorithms, efficiency differences can be interpreted as dynamical separation results. Third, we have emphasized, in particular, the importance of the computational properties of topological quantum field theories.

3. Summary of results

3.1. Quantum simulation and entanglement

3.1.1. There is a quartic entanglement invariant which measures global entanglement for any number of qubits [1].

3.1.2. Average position of a classically diffusing particle can be simulated quadratically faster with a quantum lattice gas automaton [2].

3.1.3. Quantum lattice gas models can simulate quantum ratchets [4].

3.1.4. When noise is included in these models they simulate the current reversals that are observed in quantum ratchets [9].

3.1.5. Negative index materials can be used to simulate the Klein paradox for relativistic quantum particles [15].

3.1.6. The 'concurrence of assistance' identifies capabilities and limitations to producing pure bipartite entangled states from pure tripartite entangled states and prove that it is an entanglement monotone for $(2 \times 2 \times n)$-dimensional pure states [19].

3.2. Quantum from classical separation results

3.2.1. Quantum algorithms utilizing Hamming distance information can identify a base k string in time constant in the length of the string [5].

3.2.2. Oracle problems can be understood as concept learning problems; "amplified impatient learning" is a general quantum algorithm solving this kind of problem [11].

3.2.3. Locating a template in an image is such a problem, and hence has superior quantum solution [12].

3.2.4. Finding stable models of logic programs can be achieved by a modification of Grover's algorithm [13].

3.2.5. Marinatto and Weber type quantum games should be understood as quantum communication protocols, and their outcomes can be achieved with classical communication [16].

3.2.6. The quantum logic for $\mathbb{C}^{2^{(n+1)}}$ is not equal to the quantum logic for $\mathbb{C}^{2^n}$ [17].

3.3. Topological quantum field computation

3.3.1. Theoretically, quantum computation can be implemented in anyonic systems, *i.e.*, with unitary topological modular functors [7].

3.3.2. The SU(2) Witten-Chern-Simons modular functor at an r^{th} root of unity is universal for quantum computation if $r \notin \{3, 4, 6\}$ [3].

3.3.3. Quantum SU(2) faithfully detects mapping class groups modulo center [6].

3.3.4. There are simple models of spin $\frac{1}{2}$ particles interacting on a lattice that, because of rigidity results on von Neumann algebras, are conjectured to lie in the same universality class as topological quantum field theories that are universal for quantum computation [8].

3.3.5. Numerical simulations support these conjectures [10].

3.3.6. Topological phases universal for quantum computation can occur in P, T-invariant models of 2D electron systems [14].

3.3.7. In the projective representation given by the SO(3) Witten-Reshitikhin-Turaev theory at an r^{th} root of unity for r prime and greater than 3, the image of the mapping class group of a surface of genus $g > 1$ is dense [18].

4. Refereed publications

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5. Personnel

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